Class: Bid/Ask bounce

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Administrivia

- Read sections 6.2, 6.3 and 6.5

Modelling B/A bounce

- Levels are best thought of as a martingale
- So, arIma
- So we should difference
- But, bid-ask bounce is in price space:

\[ \hat{P}_t = P_t^* + I_t S/2 \]

- where \( P_t^* \) is the real price
- \( I \) is a +1/-1 RV
- \( S \) is the spread

- So the difference is:

\[ \Delta \hat{P}_t = \Delta P_t^* + (I_t - I_{t-1}) S/2 \]

- So it looks like a MA process in terms of the \( I \)'s
• If we take $\Delta P^*_t$ as zero, it is a pure MA process.

• For fast enough times, this isn’t a bad approximation

• Work out Covariance for zero noise

With a martingale term

• What about martingale? (I.e. random walk)

• work out covariance

• Not correlation is lower since it is divided by a larger variance

Empirics

• We have a negative auto-correlation

• i.e. “mean reverting.”

• But not one we can trade on

• So useful for predictions: but not for making money

Need to model inter-arrival times (section 6.5)

• Whole area of probabilistic modelling, eg renewal theory

• Easiest model: inter-arrivals are independent

  – $W_i$ is waiting time between events
  – $T_k = \sum^k W_i$ is time until $k$th event
  – $N_t = \inf\{k|T_k > t\}$ is the count of the number of trades
  – See stat 433 for details (all of chapter 7)
• But harder in finance

• Want to allow for inhomogenous trading rates
  – Easy: more trades in morning, fewer during lunch
  – Harder: hot stocks, requires an ARCH like model

Model for duration

• First take out the easy part: \( x_i = W_i / f(T_i) \)
  – \( W_i \) is raw waiting time
  – \( T_i \) is the time we are talking about
  – \( f() \) is the basic speed (say time of day effect)

• Now estimate speed by GARCH like effect
  – \( \psi_i \) is say exponential smooth of \( x_i \)’s
  – Can build as complex an estimator as one likes

• Now model \( X_i = \psi_i \epsilon_i \)
  – Where \( \epsilon_i \) are say IID exponentials
  – Some like Wiebell, or Gamma distribution