Administrivia

Homework: Asymptotics #19.

Conditional probability as counts

- Life table
- $P(X > 80) = 57062$
- $P(X > 60) = 89835$
- $P(X > 80 \mid X > 60) = .63$ (note $> P(X > 80)$)

Story time: life tables

- How was this computed?
  - Measured how many people who were 60 in 1990 live to 61 in 1991
• So relive 1990 over and over again. Like *ground hog’s day*
• But we will all live in the future
• Rate of death is halving every 20 years.
• Do infinite sum:
  \[ P(\text{live for ever}) = (1 - p)(1 - p/2)(1 - p/4) \cdot \]
  \[ = \exp\{\sum \log(1 - p/2^i)\} \]
  \[ = \exp\{-p \sum /2^i - p^2/2 \sum 4^i\} \]
  \[ = \exp\{-p - p^2/6\} \]
  \[ > 0 \]
• Singularity (where I spent the weekend)
• Unfortunately, you will be 20 years older in 20 years.
• Probability of death for 20 years older is greater than 2*prob now.
• So we need more!

**Conditional probability definition**

\[ P(A|B) = P(A \cap B)/P(B) \]

Big difference. \( A = \) abusive husband, \( D = \) murdered woman,

\[ P(D|A) \approx 0 \]

Which is a good thing since there are lots of abusive people.

\[ P(A|D) \approx .5 \]
Bayes rule

Drug testing. Test faculty for drugs. \( D = \) drug use, \( T = \) test positive. Science tells us

\[
P(T|D) = .9
\]

Not enough! Also want to know

\[
P(\overline{T}|D) = .1
\]

Ok, that’s better. Can we compute \( P(T \cap D) \)? No. Need more.

\[
P(D) = .01
\]

So,

\[
P(D|T) = \frac{P(D \cap T)}{P(T)} = \cdots
\]

Easier with table

Monty Hall if time.

Draw it all out.

Independence

Amazingly lucky formula:

\[
P(B) = P(B|A)
\]

Wouldn’t life be nice? Called independence.
What power in a word. “Events” $A_1, A_2, A_3, \ldots, A_n$ are independent if

$$P(A_1 \cap A_2) = P(A_1)P(A_2)$$
$$P(A_1 \cap A_3) = P(A_1)P(A_3)$$
$$\vdots$$
$$P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2)P(A_3)$$
$$\vdots$$
$$P(A_1 \cap A_2 \cap A_3 \cap A_4) = P(A_1)P(A_2)P(A_3)P(A_4)$$

How many equations in all? $2^n$ or so!

If $P(A_i) = p$ which doesn’t depend on $i$, we get to compress this power even more. IID = Independent and identically distributed. Just sneak these three letters in somewhere and you have snunk in $2^n + n$ equations. Now anything is easy to compute.

**Random variables**

Create random variables out of $A_1$ and $A_2$. They are independent: $I_{A_1}$ independent of $I_{A_2}$.

In general: $X$ and $Y$ are independent if for any pair of numbers, $x, y$

$$P(X = x, Y = y) = P(X = x)P(Y = y)$$