Chapter 3.2 teaching notes

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Administrivia

Homework questions?

Game: what is the most important theorem in all of mathematics?

Game we played at the mathematics department:

- Intermediate value theorem
- Central limit theorem
- Binomial theorem
- Taylor’s theorem

Definition

- \( n \) choose \( k \)
• We are done for the day!

• Theorem: \( \binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k} \). Why?

• Theorem: \( \binom{n}{k} = \frac{n!}{k!(n-k)!} \), Why?

**Required stupid combinatorics**

• Poker: Full house, its just counting

• 13 choose 2, * 2 choose 1 * 4 choose 3 * 4 choose 2

**Binomial distribution**

• Bernulli trials

• If total is all that matters–Binomial distribution

• We can compute it

**Binomial theorem**

\[
(a + b)^n = \cdots
\]

**Inclusion exclusion proof**

**Theorem 1** Let \( P \) be a probability distribution on a sample space \( \Omega \), and let \( \{A_1, A_2, \ldots, A_n\} \) be a finite set of events. Then

\[
P(A_1 \cup A_2 \cup \cdots \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{1 \leq i < j \leq n} P(A_i \cap A_j)
\]
\[
\sum_{1 \leq i < j < k \leq n} P(A_i \cap A_j \cap A_k) - \cdots.
\]

(1)

That is, to find the probability that at least one of \(n\) events \(A_i\) occurs, first add the probability of each event, then subtract the probabilities of all possible two-way intersections, add the probability of all three-way intersections, and so forth. If the outcome \(\omega\) occurs in at least one of the events \(A_i\), its probability is added exactly once by the left side of Equation ?? . We must show that it is added exactly once by the right side of Equation ?? . Assume that \(\omega\) is in exactly \(k\) of the sets. Then its probability is added \(k\) times in the first term, subtracted \(\binom{k}{2}\) times in the second, added \(\binom{k}{3}\) times in the third term, and so forth. Thus, the total number of times that it is added is

\[
\binom{k}{1} - \binom{k}{2} + \binom{k}{3} - \cdots (-1)^{k-1} \binom{k}{k}.
\]

But

\[
0 = (1 - 1)^k = \sum_{j=0}^{k} \binom{k}{j} (-1)^j = \binom{k}{0} - \sum_{j=1}^{k} \binom{k}{j} (-1)^{j-1}.
\]

Hence,

\[
1 = \binom{k}{0} = \sum_{j=1}^{k} \binom{k}{j} (-1)^{j-1}.
\]

If the outcome \(\omega\) is not in any of the events \(A_i\), then it is not counted on either side of the equation.

Hats

Inclusion exclusion on fixed points
So why was binomial theorem my favorite?

From it you learn about

- statistics (see book)
- $e$
- finance $(1 + \epsilon)^n$
- inclusion - exclusion (I didn’t konw this one before)
- WINNER!