Here are the answer to JUST the web question. Sivan should be writing up the rest of them soon. Since I screwed up the question, I figured I had to write the ansewr. We ended up not grading the web question since it was difficult to guess what I was actually asking you to do!

- (Web problem) Actually, I used a different proof this year–so this problem was harder than I meant it to be. Further it was even more confusing that it should have been!

  1. **Prove that Y_t is a martingale.**
     $E(Y_t|\mathcal{F}_{t-1}) = 1/3(2Y_{t-1}) + 2/3(Y_{t-1}/2) = Y_{t-1}$. Since $X_t < X_0 + t$, $Y_t < 2^{X_0+t}$. Hence $E(|Y_t|) = E(Y_t) \leq 2^{X_0+t} < \infty$.
  2. **Compute $E(Y_t)$ given $Y_0 = 2^i$.**
     In more traditional language, this asks $E(Y_t|Y_0 = 2^i)$ which for any martingale will be 2^i.
  3. **Estimate the probability that Y_t has not been absorbed by time t.**
     This is the confusing one. It should have mentioned that t should be large! It should hvae said, "compute an upper bound." So here is the answer to the unasked question.
     If we ever saw n "heads" in a row, we definitely will be absorbed reguardless of where we started from. The chance of this occuring is $1 - (2/3)^n$. In order for us not to have been absorbed by time t, we have to have made it through t/n blocks without any of them being all heads. The chance of this is $(1 - (2/3)^n)^{t/n}$. So an upper bound on the chance of being absorbed is $1 - (1 - (2/3)^n)^{t/n}$.
  4. **How large does t have to be so that we are sure that the probability of not being absorbed is less than 2^{-n}?**
     Picking t equal to $n(3/2)^n$ will make $(1 - (2/3)^n)^{t/n}$ less than $1/e$. So making t bigger than $n^2(3/2)^n$ will achieve the desired accuracy.
  5. **Estimate $P(Y_t = 1|Y_0 = 2^i)$**
     From the equation for $E(Y_t = 1|Y_0 = 2^i)$ and the fact that there is very little chance that Y_t isn't either 1 or 2^n we get the same formula for gambler ruin as is in the book. THis clearly answer the next question also.
  6. **Does this match the formula given in the book?**
     Yup! exactly.